1 Question 1

The first problem is fairly trivial. The speed of the velociraptor is given by

$$S = 4 * t \qquad \forall t < 25/4$$

$$S = 25 \qquad \forall t \geq 25/4$$

we can use this to derive the location of the raptor, assuming R(0) = 0.

$$R(T) = \int_0^T 4tdt \qquad \forall T < 25/4$$

$$R(T) = \int_0^{25/4} 4tdt + \int_{25/4}^T 25dt \qquad \forall T \geq 25/4$$

The location of the person is given by

$$P(T) = \int_0^T 6dt + P(0) = 6T + 40$$

Equating the previous with R(T) would give us an equation for the time it takes for the raptor to catch the person. We'll assume that T is greater than 25/4 when we're calculating because if the solution gives a T less than 25/4, we'll just go back and assume that T is less than 25/4.

$$P(T) = 6T + 40 = R(T) = \int_0^{25/4} 4tdt + \int_{25/4}^T 25dt$$

$$6T + 40 = \int_0^{25/4} 4tdt + \int_{25/4}^T 25dt = 625/8 - 25((25/4) - T)$$

$$25T - 6T = 40 - 625/8 + 625/4$$

$$T = 945/152 = 6.217$$

Since $6.217 < 6.25$, we'll have to recheck our answer. Instead, we use the following equation:

$$P(T) = 6T + 40 = R(T) = \int_0^T 4t dt$$

$$6T + 40 = \frac{4T^2}{2}$$

$$4T^2 + 6T + 4 = 0$$

Take the positive solution from the binomial formula, $T = \frac{3+\sqrt{89}}{2} \approx 6.217$ which is pretty much what we got before.

So at 6.217 seconds, the raptor catches the person. The person runs about 6.217*6 = 37.3 meters before he is caught.

2 Question 2

The problem is solved using a numeric diff eq. solver and a numeric maximizer. Since the problem is symmetric along y, only the top and right side raptors were considered, and the angle is restricted to 0 to 90 degrees. For a given angle, find the least time T when the location of 1 raptor converge on the person's. This could be done by numerically solving the differential equations describing the location of the raptors, and equating that to the location of the person. So for a given angle, ϕ, we have a function $F(\phi) \rightarrow \Re$ that gives the time T. We then use newton's method or any numeric maximizer to find $\arg\max F(\phi)$.

The system of equations for the location of the raptor R1 is described by the following:

$$R1x'(t) = S1(t)\cos(\theta)$$

$$R1y'(t) = S1(t)\sin(\theta)$$

where theta describes the angle of the vector $R1 \rightarrow P$.

Subsitute $\cos(\theta)$ and $\sin(\theta)$ with appropriate computation, i.e.

$$\cos(\theta) = \frac{Px(t) - R1x(t)}{\sqrt{(Px(t) - R1x(t))^2 + (Py(t) - R1y(t))^2}}$$

$$\sin(\theta) = \frac{Py(t) - R1y(t)}{\sqrt{(Px(t) - R1x(t))^2 + (Py(t) - R1y(t))^2}}$$

The boundary condition of the raptor R1 is given by:

$$R1x(0) = 0$$

$$R1y(0) = 20\sqrt{3}/9$$

S1(t) is the speed of the raptor described by part 1, i.e.

$$S1(t) = 4t \qquad \forall t < 10/4$$

$$S1(t) = 10 \qquad \forall t \geq 10/4$$

Px(t) and Py(t) are the location of the person based on the initial angle ϕ. i.e. $(6t\cos(\phi), 6t\sin(\phi))$

R2 is described in the same way except with different boundary condition and speed.

For a given ϕ, we can solve the above differential equations numerically, and thereby obtain a T s.t. the location of 1 raptor converges with the person's. This is usually the largest number that is still in the domain of the solved functions due to the fact that the denominator (the distance between R and P) goes to 0 afterward.

Having obtained a numerical function $F(\phi) \rightarrow \Re$ for the minimum time one of the raptors converges on to the person, we can then use any maximizer to give a solution to the problem $\arg\max F(\phi)$.

The last time I ran an algorithm to solve the question, it gave about 33.00 degrees north of the horizon with the point of contact at t= 2.9 seconds. Due to the symmetry, the solutions are 33.00, and 147.00 degrees.